

4

Motion in Two Dimensions

CHAPTER OUTLINE

- 4.1 The Position, Velocity, and Acceleration Vectors
- 4.2 Two-Dimensional Motion with Constant Acceleration
- 4.3 Projectile Motion
- 4.4 Uniform Circular Motion
- 4.5 Tangential and Radial Acceleration
- 4.6 Relative Velocity and Relative Acceleration

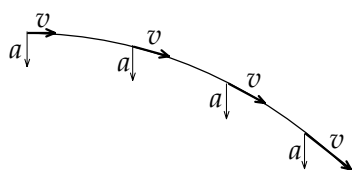
ANSWERS TO QUESTIONS

- Q4.1** Yes. An object moving in uniform circular motion moves at a constant speed, but changes its direction of motion. An object cannot accelerate if its velocity is constant.
- Q4.2** No, you cannot determine the instantaneous velocity. Yes, you can determine the average velocity. The points could be widely separated. In this case, you can only determine the average velocity, which is

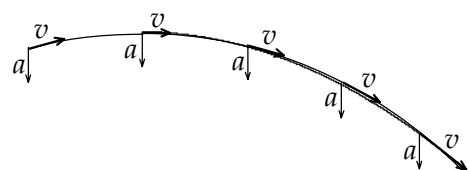
$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t}.$$

Q4.3

(a)



(b)



Q4.4

(a)

$$10\hat{\mathbf{i}} \text{ m/s}$$

(b)

$$-9.80\hat{\mathbf{j}} \text{ m/s}^2$$

Q4.5

The easiest way to approach this problem is to determine acceleration first, velocity second and finally position.

Vertical: In free flight, $a_y = -g$. At the top of a projectile's trajectory, $v_y = 0$. Using this, the maximum height can be found using $v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)$.

Horizontal: $a_x = 0$, so v_x is always the same. To find the horizontal position at maximum height, one needs the flight time, t . Using the vertical information found previously, the flight time can be found using $v_{fy} = v_{iy} + a_y t$. The horizontal position is $x_f = v_{ix} t$.

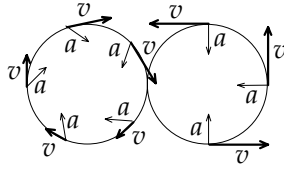
If air resistance is taken into account, then the acceleration in both the x and y -directions would have an additional term due to the drag.

Q4.6

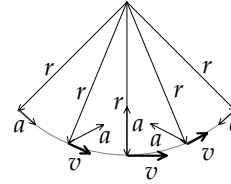
A parabola.

- Q4.7** The balls will be closest together as the second ball is thrown. Yes, the first ball will always be moving faster, since its flight time is larger, and thus the vertical component of the velocity is larger. The time interval will be one second. No, since the vertical component of the motion determines the flight time.
- Q4.8** The ball will have the greater speed. Both the rock and the ball will have the same vertical component of the velocity, but the ball will have the additional horizontal component.
- Q4.9** (a) yes (b) no (c) no (d) yes (e) no
- Q4.10** Straight up. Throwing the ball any other direction than straight up will give a nonzero speed at the top of the trajectory.
- Q4.11** No. The projectile with the larger vertical component of the initial velocity will be in the air longer.
- Q4.12** The projectile is in free fall. Its vertical component of acceleration is the downward acceleration of gravity. Its horizontal component of acceleration is zero.
- Q4.13** (a) no (b) yes (c) yes (d) no
- Q4.14** 60° . The projection angle appears in the expression for horizontal range in the function $\sin 2\theta$. This function is the same for 30° and 60° .
- Q4.15** The optimal angle would be less than 45° . The longer the projectile is in the air, the more that air resistance will change the components of the velocity. Since the vertical component of the motion determines the flight time, an angle less than 45° would increase range.
- Q4.16** The projectile on the moon would have both the larger range and the greater altitude. *Apollo* astronauts performed the experiment with golf balls.
- Q4.17** Gravity only changes the vertical component of motion. Since both the coin and the ball are falling from the same height with the same vertical component of the initial velocity, they must hit the floor at the same time.
- Q4.18** (a) no (b) yes
- In the second case, the particle is continuously changing the direction of its velocity vector.
- Q4.19** The racing car rounds the turn at a constant *speed* of 90 miles per hour.
- Q4.20** The acceleration cannot be zero because the pendulum does not remain at rest at the end of the arc.
- Q4.21** (a) The velocity is not constant because the object is constantly changing the direction of its motion.
- (b) The acceleration is not constant because the acceleration always points towards the center of the circle. The magnitude of the acceleration is constant, but not the direction.
- Q4.22** (a) straight ahead (b) in a circle or straight ahead

Q4.23



Q4.24



Q4.25 The unit vectors $\hat{r}$ and $\hat{\theta}$ are in different directions at different points in the xy plane. At a location along the x -axis, for example, $\hat{r} = \hat{i}$ and $\hat{\theta} = \hat{j}$, but at a point on the y -axis, $\hat{r} = \hat{j}$ and $\hat{\theta} = -\hat{i}$. The unit vector $\hat{i}$ is equal everywhere, and $\hat{j}$ is also uniform.

Q4.26 The wrench will hit at the base of the mast. If air resistance is a factor, it will hit slightly leeward of the base of the mast, displaced in the direction in which air is moving relative to the deck. If the boat is scudding before the wind, for example, the wrench's impact point can be in front of the mast.

Q4.27 (a) The ball would move straight up and down as observed by the passenger. The ball would move in a parabolic trajectory as seen by the ground observer.

(b) Both the passenger and the ground observer would see the ball move in a parabolic trajectory, although the two observed paths would not be the same.

Q4.28 (a) g downward

(b) g downward

The horizontal component of the motion does not affect the vertical acceleration.

SOLUTIONS TO PROBLEMS

Section 4.1 The Position, Velocity, and Acceleration Vectors

P4.1

$x(m)$	$y(m)$
0	-3 600
-3 000	0
-1 270	1 270
-4 270 m	-2 330 m

$$(a) \quad \text{Net displacement} = \sqrt{x^2 + y^2} = \boxed{4.87 \text{ km at } 28.6^\circ \text{ S of W}}$$

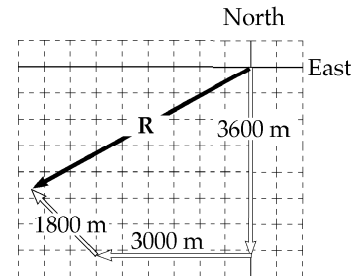


FIG. P4.1

$$(b) \quad \text{Average speed} = \frac{(20.0 \text{ m/s})(180 \text{ s}) + (25.0 \text{ m/s})(120 \text{ s}) + (30.0 \text{ m/s})(60.0 \text{ s})}{180 \text{ s} + 120 \text{ s} + 60.0 \text{ s}} = \boxed{23.3 \text{ m/s}}$$

$$(c) \quad \text{Average velocity} = \frac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along R}}$$

- P4.2**
- (a) $\mathbf{r} = \boxed{18.0t\hat{\mathbf{i}} + (4.00t - 4.90t^2)\hat{\mathbf{j}}}$
- (b) $\mathbf{v} = \boxed{(18.0 \text{ m/s})\hat{\mathbf{i}} + [4.00 \text{ m/s} - (9.80 \text{ m/s}^2)t]\hat{\mathbf{j}}}$
- (c) $\mathbf{a} = \boxed{(-9.80 \text{ m/s}^2)\hat{\mathbf{j}}}$
- (d) $\mathbf{r}(3.00 \text{ s}) = \boxed{(54.0 \text{ m})\hat{\mathbf{i}} - (32.1 \text{ m})\hat{\mathbf{j}}}$
- (e) $\mathbf{v}(3.00 \text{ s}) = \boxed{(18.0 \text{ m/s})\hat{\mathbf{i}} - (25.4 \text{ m/s})\hat{\mathbf{j}}}$
- (f) $\mathbf{a}(3.00 \text{ s}) = \boxed{(-9.80 \text{ m/s}^2)\hat{\mathbf{j}}}$

***P4.3** The sun projects onto the ground the x -component of her velocity:

$$5.00 \text{ m/s} \cos(-60.0^\circ) = \boxed{2.50 \text{ m/s}}.$$

P4.4 (a) From $x = -5.00 \sin \omega t$, the x -component of velocity is

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt} \right) (-5.00 \sin \omega t) = -5.00\omega \cos \omega t$$

$$\text{and } a_x = \frac{dv_x}{dt} = +5.00\omega^2 \sin \omega t$$

$$\text{similarly, } v_y = \left(\frac{d}{dt} \right) (4.00 - 5.00 \cos \omega t) = 0 + 5.00\omega \sin \omega t$$

$$\text{and } a_y = \left(\frac{d}{dt} \right) (5.00\omega \sin \omega t) = 5.00\omega^2 \cos \omega t.$$

$$\text{At } t = 0, \mathbf{v} = -5.00\omega \cos 0\hat{\mathbf{i}} + 5.00\omega \sin 0\hat{\mathbf{j}} = \boxed{(5.00\omega\hat{\mathbf{i}} + 0\hat{\mathbf{j}}) \text{ m/s}}$$

$$\text{and } \mathbf{a} = 5.00\omega^2 \sin 0\hat{\mathbf{i}} + 5.00\omega^2 \cos 0\hat{\mathbf{j}} = \boxed{(0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}) \text{ m/s}^2}.$$

(b) $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \boxed{(4.00 \text{ m})\hat{\mathbf{j}} + (5.00 \text{ m})(-\sin \omega t\hat{\mathbf{i}} - \cos \omega t\hat{\mathbf{j}})}$

$$\mathbf{v} = \boxed{(5.00 \text{ m})\omega[-\cos \omega t\hat{\mathbf{i}} + \sin \omega t\hat{\mathbf{j}}]}$$

$$\mathbf{a} = \boxed{(5.00 \text{ m})\omega^2[\sin \omega t\hat{\mathbf{i}} + \cos \omega t\hat{\mathbf{j}}]}$$

(c) The object moves in $\boxed{\text{a circle of radius } 5.00 \text{ m centered at } (0, 4.00 \text{ m})}.$

Section 4.2 Two-Dimensional Motion with Constant Acceleration

P4.5 (a) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t$

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t} = \frac{(9.00\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}) - (3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})}{3.00} = \boxed{(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}) \text{ m/s}^2}$$

(b) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2} \mathbf{a} t^2 = (3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})t + \frac{1}{2}(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})t^2$

$$\boxed{x = (3.00t + t^2) \text{ m}} \quad \text{and} \quad \boxed{y = (1.50t^2 - 2.00t) \text{ m}}$$

P4.6 (a) $\mathbf{v} = \frac{d\mathbf{r}}{dt} = \left(\frac{d}{dt}\right)(3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}) = \boxed{-12.0t\hat{\mathbf{j}} \text{ m/s}}$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \left(\frac{d}{dt}\right)(-12.0t\hat{\mathbf{j}}) = \boxed{-12.0\hat{\mathbf{j}} \text{ m/s}^2}$$

(b) $\boxed{\mathbf{r} = (3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}) \text{ m}; \mathbf{v} = -12.0\hat{\mathbf{j}} \text{ m/s}}$

P4.7 $\mathbf{v}_i = (4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}) \text{ m/s}$ and $\mathbf{v}(20.0) = (20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}) \text{ m/s}$

(a) $a_x = \frac{\Delta v_x}{\Delta t} = \frac{20.0 - 4.00}{20.0} \text{ m/s}^2 = \boxed{0.800 \text{ m/s}^2}$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-5.00 - 1.00}{20.0} \text{ m/s}^2 = \boxed{-0.300 \text{ m/s}^2}$$

(b) $\theta = \tan^{-1}\left(\frac{-0.300}{0.800}\right) = -20.6^\circ = \boxed{339^\circ \text{ from } +x \text{ axis}}$

(c) At $t = 25.0 \text{ s}$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 10.0 + 4.00(25.0) + \frac{1}{2}(0.800)(25.0)^2 = \boxed{360 \text{ m}}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = -4.00 + 1.00(25.0) + \frac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7 \text{ m}}$$

$$v_{xf} = v_{xi} + a_x t = 4 + 0.8(25) = 24 \text{ m/s}$$

$$v_{yf} = v_{yi} + a_y t = 1 - 0.3(25) = -6.5 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2^\circ}$$

84 Motion in Two Dimensions

P4.8 $\mathbf{a} = 3.00\hat{\mathbf{j}} \text{ m/s}^2$; $\mathbf{v}_i = 5.00\hat{\mathbf{i}} \text{ m/s}$; $\mathbf{r}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$

(a) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2} \mathbf{a} t^2 = \boxed{\left[5.00t\hat{\mathbf{i}} + \frac{1}{2} 3.00t^2\hat{\mathbf{j}} \right] \text{ m}}$

$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a} t = \boxed{(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}) \text{ m/s}}$

(b) $t = 2.00 \text{ s}$, $\mathbf{r}_f = 5.00(2.00)\hat{\mathbf{i}} + \frac{1}{2}(3.00)(2.00)^2\hat{\mathbf{j}} = (10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}) \text{ m}$

so $x_f = \boxed{10.0 \text{ m}}$, $y_f = \boxed{6.00 \text{ m}}$

$\mathbf{v}_f = 5.00\hat{\mathbf{i}} + 3.00(2.00)\hat{\mathbf{j}} = (5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}) \text{ m/s}$

$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81 \text{ m/s}}$

***P4.9** (a) For the x -component of the motion we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$.

$$\begin{aligned} 0.01 \text{ m} &= 0 + (1.80 \times 10^7 \text{ m/s})t + \frac{1}{2}(8 \times 10^{14} \text{ m/s}^2)t^2 \\ (4 \times 10^{14} \text{ m/s}^2)t^2 + (1.80 \times 10^7 \text{ m/s})t - 10^{-2} \text{ m} &= 0 \\ t &= \frac{-1.80 \times 10^7 \text{ m/s} \pm \sqrt{(1.80 \times 10^7 \text{ m/s})^2 - 4(4 \times 10^{14} \text{ m/s}^2)(-10^{-2} \text{ m})}}{2(4 \times 10^{14} \text{ m/s}^2)} \\ &= \frac{-1.8 \times 10^7 \pm 1.84 \times 10^7 \text{ m/s}}{8 \times 10^{14} \text{ m/s}^2} \end{aligned}$$

We choose the $+$ sign to represent the physical situation

$$t = \frac{4.39 \times 10^5 \text{ m/s}}{8 \times 10^{14} \text{ m/s}^2} = 5.49 \times 10^{-10} \text{ s}.$$

Here

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{1}{2}(1.6 \times 10^{15} \text{ m/s}^2)(5.49 \times 10^{-10} \text{ s})^2 = 2.41 \times 10^{-4} \text{ m}.$$

So, $\mathbf{r}_f = \boxed{(10.0\hat{\mathbf{i}} + 0.241\hat{\mathbf{j}}) \text{ mm}}$.

(b) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = 1.80 \times 10^7 \text{ m/s}\hat{\mathbf{i}} + (8 \times 10^{14} \text{ m/s}^2\hat{\mathbf{i}} + 1.6 \times 10^{15} \text{ m/s}^2\hat{\mathbf{j}})(5.49 \times 10^{-10} \text{ s})$
 $= (1.80 \times 10^7 \text{ m/s})\hat{\mathbf{i}} + (4.39 \times 10^5 \text{ m/s})\hat{\mathbf{i}} + (8.78 \times 10^5 \text{ m/s})\hat{\mathbf{j}}$
 $= \boxed{(1.84 \times 10^7 \text{ m/s})\hat{\mathbf{i}} + (8.78 \times 10^5 \text{ m/s})\hat{\mathbf{j}}}$

(c) $|\mathbf{v}_f| = \sqrt{(1.84 \times 10^7 \text{ m/s})^2 + (8.78 \times 10^5 \text{ m/s})^2} = \boxed{1.85 \times 10^7 \text{ m/s}}$

(d) $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{8.78 \times 10^5}{1.84 \times 10^7}\right) = \boxed{2.73^\circ}$

Section 4.3 Projectile Motion

P4.10 $x = v_{xi}t = v_i \cos \theta_i t$

$$x = (300 \text{ m/s})(\cos 55.0^\circ)(42.0 \text{ s})$$

$$x = \boxed{7.23 \times 10^3 \text{ m}}$$

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin \theta_i t - \frac{1}{2}gt^2$$

$$y = (300 \text{ m/s})(\sin 55.0^\circ)(42.0 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(42.0 \text{ s})^2 = \boxed{1.68 \times 10^3 \text{ m}}$$

- P4.11** (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$, i.e.,

$$t = \frac{x_f}{v_{xi}} = \frac{(1.40 \text{ m})}{v_{xi}}$$

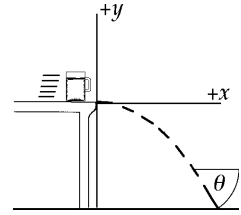


FIG. P4.11

In the same time it falls a distance of 0.860 m with acceleration downward of 9.80 m/s^2 . Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2: 0 = 0.860 \text{ m} + \frac{1}{2}(-9.80 \text{ m/s}^2)\left(\frac{1.40 \text{ m}}{v_{xi}}\right)^2.$$

Thus,

$$v_{xi} = \sqrt{\frac{(4.90 \text{ m/s}^2)(1.96 \text{ m}^2)}{0.860 \text{ m}}} = \boxed{3.34 \text{ m/s}}.$$

- (b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + (-9.80 \text{ m/s}^2)\left(\frac{1.40 \text{ m}}{3.34 \text{ m/s}}\right) = -4.11 \text{ m/s}.$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9^\circ}.$$

- P4.12** The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x_f = v_{xi} t + \frac{1}{2} a_x t^2 = v_{xi} t + 0 \quad \text{and} \quad y_f = v_{yi} t + \frac{1}{2} a_y t^2 = 0 - \frac{1}{2} g t^2.$$

When the mug reaches the floor, $y_f = -h$ so

$$-h = -\frac{1}{2} g t^2$$

which gives the time of impact as

$$t = \sqrt{\frac{2h}{g}}.$$

- (a) Since $x_f = d$ when the mug reaches the floor, $x_f = v_{xi} t$ becomes $d = v_{xi} \sqrt{\frac{2h}{g}}$ giving the initial velocity as

$$v_{xi} = d \sqrt{\frac{g}{2h}}.$$

- (b) Just before impact, the x -component of velocity is still

$$v_{xf} = v_{xi}$$

while the y -component is

$$v_{yf} = v_{yi} + a_y t = 0 - g \sqrt{\frac{2h}{g}}.$$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1} \left(\frac{|v_{yf}|}{v_{xf}} \right) = \tan^{-1} \left(\frac{g \sqrt{\frac{2h}{g}}}{d \sqrt{\frac{g}{2h}}} \right) = \tan^{-1} \left(\frac{2h}{d} \right).$$

- P4.13** (a) The time of flight of the first snowball is the nonzero root of $y_f = y_i + v_{yi}t_1 + \frac{1}{2}a_y t_1^2$

$$0 = 0 + (25.0 \text{ m/s})(\sin 70.0^\circ)t_1 - \frac{1}{2}(9.80 \text{ m/s}^2)t_1^2$$

$$t_1 = \frac{2(25.0 \text{ m/s})\sin 70.0^\circ}{9.80 \text{ m/s}^2} = 4.79 \text{ s}.$$

The distance to your target is

$$x_f - x_i = v_{xi}t_1 = (25.0 \text{ m/s})\cos 70.0^\circ(4.79 \text{ s}) = 41.0 \text{ m}.$$

Now the second snowball we describe by

$$y_f = y_i + v_{yi}t_2 + \frac{1}{2}a_y t_2^2$$

$$0 = (25.0 \text{ m/s})\sin \theta_2 t_2 - (4.90 \text{ m/s}^2)t_2^2$$

$$t_2 = (5.10 \text{ s})\sin \theta_2$$

$$x_f - x_i = v_{xi} t_2$$

$$41.0 \text{ m} = (25.0 \text{ m/s})\cos \theta_2 (5.10 \text{ s})\sin \theta_2 = (128 \text{ m})\sin \theta_2 \cos \theta_2$$

$$0.321 = \sin \theta_2 \cos \theta_2$$

Using $\sin 2\theta = 2 \sin \theta \cos \theta$ we can solve $0.321 = \frac{1}{2} \sin 2\theta_2$

$$2\theta_2 = \sin^{-1} 0.643 \text{ and } \theta_2 = \boxed{20.0^\circ}.$$

- (b) The second snowball is in the air for time $t_2 = (5.10 \text{ s})\sin \theta_2 = (5.10 \text{ s})\sin 20^\circ = 1.75 \text{ s}$, so you throw it after the first by

$$t_1 - t_2 = 4.79 \text{ s} - 1.75 \text{ s} = \boxed{3.05 \text{ s}}.$$

- P4.14** From Equation 4.14 with $R = 15.0 \text{ m}$, $v_i = 3.00 \text{ m/s}$, $\theta_{\max} = 45.0^\circ$

$$\therefore g = \frac{v_i^2}{R} = \frac{9.00}{15.0} = \boxed{0.600 \text{ m/s}^2}$$

88 Motion in Two Dimensions

P4.15 $h = \frac{v_i^2 \sin^2 \theta_i}{2g}; R = \frac{v_i^2 (\sin 2\theta_i)}{g}; 3h = R,$
 so $\frac{3v_i^2 \sin^2 \theta_i}{2g} = \frac{v_i^2 (\sin 2\theta_i)}{g}$
 or $\frac{2}{3} = \frac{\sin^2 \theta_i}{\sin 2\theta_i} = \frac{\tan \theta_i}{2}$
 thus $\theta_i = \tan^{-1}\left(\frac{4}{3}\right) = \boxed{53.1^\circ}.$

***P4.16** (a) To identify the maximum height we let i be the launch point and f be the highest point:

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y(y_f - y_i) \\ 0 &= v_i^2 \sin^2 \theta_i + 2(-g)(y_{\max} - 0) \\ y_{\max} &= \frac{v_i^2 \sin^2 \theta_i}{2g}. \end{aligned}$$

To identify the range we let i be the launch and f be the impact point; where t is not zero:

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_yt^2 \\ 0 &= 0 + v_i \sin \theta_i t + \frac{1}{2}(-g)t^2 \\ t &= \frac{2v_i \sin \theta_i}{g} \\ x_f &= x_i + v_{xi}t + \frac{1}{2}a_xt^2 \\ d &= 0 + v_i \cos \theta_i \frac{2v_i \sin \theta_i}{g} + 0. \end{aligned}$$

For this rock, $d = y_{\max}$

$$\begin{aligned} \frac{v_i^2 \sin^2 \theta_i}{2g} &= \frac{2v_i^2 \sin \theta_i \cos \theta_i}{g} \\ \frac{\sin \theta_i}{\cos \theta_i} &= \tan \theta_i = 4 \\ \theta_i &= \boxed{76.0^\circ} \end{aligned}$$

(b) Since g divides out, the answer is the same on every planet.

(c) The maximum range is attained for $\theta_i = 45^\circ$:

$$\frac{d_{\max}}{d} = \frac{v_i \cos 45^\circ 2v_i \sin 45^\circ g}{g v_i \cos 76^\circ 2v_i \sin 76^\circ} = 2.125.$$

So $d_{\max} = \boxed{\frac{17d}{8}}.$

P4.17 (a) $x_f = v_{xi} t = 8.00 \cos 20.0^\circ (3.00) = \boxed{22.6 \text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi} t + \frac{1}{2} g t^2$$

$$y_f = 8.00 \sin 20.0^\circ (3.00) + \frac{1}{2} (9.80) (3.00)^2 = \boxed{52.3 \text{ m}}.$$

(c) $10.0 = 8.00(\sin 20.0^\circ)t + \frac{1}{2}(9.80)t^2$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18 \text{ s}}$$

***P4.18** We interpret the problem to mean that the displacement from fish to bug is

$$2.00 \text{ m at } 30^\circ = (2.00 \text{ m})\cos 30^\circ \hat{\mathbf{i}} + (2.00 \text{ m})\sin 30^\circ \hat{\mathbf{j}} = (1.73 \text{ m})\hat{\mathbf{i}} + (1.00 \text{ m})\hat{\mathbf{j}}.$$

If the water should drop 0.03 m during its flight, then the fish must aim at a point 0.03 m above the bug. The initial velocity of the water then is directed through the point with displacement

$$(1.73 \text{ m})\hat{\mathbf{i}} + (1.03 \text{ m})\hat{\mathbf{j}} = 2.015 \text{ m at } 30.7^\circ.$$

For the time of flight of a water drop we have

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$1.73 \text{ m} = 0 + (v_i \cos 30.7^\circ)t + 0 \text{ so}$$

$$t = \frac{1.73 \text{ m}}{v_i \cos 30.7^\circ}.$$

The vertical motion is described by

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2.$$

The “drop on its path” is

$$-3.00 \text{ cm} = \frac{1}{2} (-9.80 \text{ m/s}^2) \left(\frac{1.73 \text{ m}}{v_i \cos 30.7^\circ} \right)^2.$$

Thus,

$$v_i = \frac{1.73 \text{ m}}{\cos 30.7^\circ} \sqrt{\frac{9.80 \text{ m/s}^2}{2 \times 0.03 \text{ m}}} = 2.015 \text{ m} (12.8 \text{ s}^{-1}) = \boxed{25.8 \text{ m/s}}.$$

P4.19 (a) We use the trajectory equation:

$$y_f = x_f \tan \theta_i - \frac{gx_f^2}{2v_i^2 \cos^2 \theta_i}.$$

With

$$x_f = 36.0 \text{ m}, v_i = 20.0 \text{ m/s}, \text{ and } \theta = 53.0^\circ$$

we find

$$y_f = (36.0 \text{ m}) \tan 53.0^\circ - \frac{(9.80 \text{ m/s}^2)(36.0 \text{ m})^2}{2(20.0 \text{ m/s})^2 \cos^2(53.0^\circ)} = 3.94 \text{ m}.$$

The ball clears the bar by

$$(3.94 - 3.05) \text{ m} = \boxed{0.889 \text{ m}}.$$

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin \theta_i}{g} = \frac{(20.0 \text{ m/s})(\sin 53.0^\circ)}{9.80 \text{ m/s}^2} = 1.63 \text{ s}.$$

The time to travel 36.0 m horizontally is $t_2 = \frac{x_f}{v_{ix}}$

$$t_2 = \frac{36.0 \text{ m}}{(20.0 \text{ m/s})(\cos 53.0^\circ)} = 2.99 \text{ s}.$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}.$

P4.20 The horizontal component of displacement is $x_f = v_{xi}t = (v_i \cos \theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \frac{d}{v_i \cos \theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \sin \theta_i \left(\frac{d}{v_i \cos \theta_i} \right) - \frac{g}{2} \left(\frac{d}{v_i \cos \theta_i} \right)^2.$$

Therefore the water strikes the building at a height h above ground level of

$$h = y_f = \boxed{d \tan \theta_i - \frac{gd^2}{2v_i^2 \cos^2 \theta_i}}.$$

- *P4.21** (a) For the horizontal motion, we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$24 \text{ m} = 0 + v_i(\cos 53^\circ)(2.2 \text{ s}) + 0$$

$$v_i = \boxed{18.1 \text{ m/s}}.$$

- (b) As it passes over the wall, the ball is above the street by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$y_f = 0 + (18.1 \text{ m/s})(\sin 53^\circ)(2.2 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(2.2 \text{ s})^2 = 8.13 \text{ m}.$$

So it clears the parapet by $8.13 \text{ m} - 7 \text{ m} = \boxed{1.13 \text{ m}}.$

- (c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation

$$y_f = (\tan \theta_i)x_f - \left(\frac{g}{2v_i^2 \cos^2 \theta_i} \right)x_f^2$$

or

$$6 \text{ m} = (\tan 53^\circ)x_f - \left(\frac{9.8 \text{ m/s}^2}{2(18.1 \text{ m/s})^2 \cos^2 53^\circ} \right)x_f^2.$$

Solving,

$$(0.0412 \text{ m}^{-1})x_f^2 - 1.33x_f + 6 \text{ m} = 0$$

and

$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.0412)(6)}}{2(0.0412 \text{ m}^{-1})}.$$

This yields two results:

$$x_f = 26.8 \text{ m or } 5.44 \text{ m}$$

The ball passes twice through the level of the roof.

It hits the roof at distance from the wall

$$26.8 \text{ m} - 24 \text{ m} = \boxed{2.79 \text{ m}}.$$

***P4.22** When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$\begin{aligned}
 x_f &= \sqrt{(3.25 \text{ km})^2 - (2.15 \text{ km})^2} = 2.437 \text{ km} \\
 y_f &= x_f \tan \theta - \frac{gx_f^2}{2v_i^2 \cos^2 \theta_i} \\
 -2150 \text{ m} &= (2437 \text{ m}) \tan \theta_i - \frac{(9.8 \text{ m/s}^2)(2437 \text{ m})^2}{2(280 \text{ m/s})^2 \cos^2 \theta_i} \\
 \therefore -2150 \text{ m} &= (2437 \text{ m}) \tan \theta_i - (371.19 \text{ m})(1 + \tan^2 \theta_i) \\
 \therefore \tan^2 \theta - 6.565 \tan \theta_i - 4.792 &= 0 \\
 \therefore \tan \theta_i &= \frac{1}{2} \left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)} \right) = 3.283 \pm 3.945.
 \end{aligned}$$

Select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan \theta_i = -0.662, \quad \boxed{\theta_i = -33.5^\circ}$$

P4.23 The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from

$$\begin{aligned}
 y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \\
 -40.0 \text{ m} &= 0 + 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2 \\
 t &= 2.86 \text{ s}.
 \end{aligned}$$

The extra time $3.00 \text{ s} - 2.86 \text{ s} = 0.143 \text{ s}$ is the time required for the sound she hears to travel straight back to the player.

It covers distance

$$(343 \text{ m/s})(0.143 \text{ s}) = 49.0 \text{ m} = \sqrt{x^2 + (40.0 \text{ m})^2}$$

where x represents the horizontal distance the rock travels.

$$\begin{aligned}
 x &= 28.3 \text{ m} = v_{xi}t + 0t^2 \\
 \therefore v_{xi} &= \frac{28.3 \text{ m}}{2.86 \text{ s}} = \boxed{9.91 \text{ m/s}}
 \end{aligned}$$

P4.24 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80 \text{ m/s}^2)(1.85 - 1.02) \text{ m}$. From this, $v_{yi} = 4.03 \text{ m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80 \text{ m/s}^2)(0.900 - 1.85) \text{ m}$. Thus the vertical velocity just before he lands is

$$v_{yf} = -4.32 \text{ m/s}.$$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32 \text{ m/s} = 4.03 \text{ m/s} + (-9.80 \text{ m/s}^2)t$$

$$\text{or } t = \boxed{0.852 \text{ s}}.$$

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80 \text{ m} = v_{xi}(0.852 \text{ s})$$

$$\text{which yields } v_{xi} = \boxed{3.29 \text{ m/s}}.$$

(c) $v_{yi} = \boxed{4.03 \text{ m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\frac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\frac{4.03 \text{ m/s}}{3.29 \text{ m/s}}\right) = \boxed{50.8^\circ}$.

(e) Similarly for the deer, the upward part of the flight gives

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i):$$

$$0 = v_{yi}^2 + 2(-9.80 \text{ m/s}^2)(2.50 - 1.20) \text{ m}$$

$$\text{so } v_{yi} = 5.04 \text{ m/s}.$$

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields $v_{yf}^2 = 0 + 2(-9.80 \text{ m/s}^2)(0.700 - 2.50) \text{ m}$ and $v_{yf} = -5.94 \text{ m/s}$.

The hang time is then found as $v_{yf} = v_{yi} + a_y t$: $-5.94 \text{ m/s} = 5.04 \text{ m/s} + (-9.80 \text{ m/s}^2)t$ and

$$\boxed{t = 1.12 \text{ s}}.$$

***P4.25** The arrow's flight time to the collision point is

$$t = \frac{x_f - x_i}{v_{xi}} = \frac{150 \text{ m}}{(45 \text{ m/s}) \cos 50^\circ} = 5.19 \text{ s}.$$

The arrow's altitude at the collision is

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \\ &= 0 + (45 \text{ m/s})(\sin 50^\circ)5.19 \text{ s} + \frac{1}{2}(-9.8 \text{ m/s}^2)(5.19 \text{ s})^2 = 47.0 \text{ m}. \end{aligned}$$

(a) The required launch speed for the apple is given by

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y(y_f - y_i) \\ 0 &= v_{yi}^2 + 2(-9.8 \text{ m/s}^2)(47 \text{ m} - 0) \\ v_{yi} &= \boxed{30.3 \text{ m/s}}. \end{aligned}$$

(b) The time of flight of the apple is given by

$$\begin{aligned} v_{yf} &= v_{yi} + a_y t \\ 0 &= 30.3 \text{ m/s} - 9.8 \text{ m/s}^2 t \\ t &= 3.10 \text{ s}. \end{aligned}$$

So the apple should be launched after the arrow by $5.19 \text{ s} - 3.10 \text{ s} = \boxed{2.09 \text{ s}}$.

***P4.26** For the smallest impact angle

$$\theta = \tan^{-1} \left(\frac{v_{yf}}{v_{xf}} \right),$$

we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y -component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi} . Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v$, $v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is

$$\theta = \tan^{-1} \left(\frac{v_{yf}}{v_{xf}} \right) = \boxed{\tan^{-1} \left(\frac{\sqrt{2gh}}{v} \right)}.$$

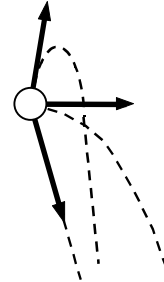


FIG. P4.26

Section 4.4 Uniform Circular Motion

P4.27 $a_c = \frac{v^2}{r} = \frac{(20.0 \text{ m/s})^2}{1.06 \text{ m}} = \boxed{377 \text{ m/s}^2}$

The mass is unnecessary information.

P4.28 $a = \frac{v^2}{R}, T = 24 \text{ h}(3600 \text{ s/h}) = 86400 \text{ s}$
 $v = \frac{2\pi R}{T} = \frac{2\pi(6.37 \times 10^6 \text{ m})}{86400 \text{ s}} = 463 \text{ m/s}$
 $a = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}} = \boxed{0.0337 \text{ m/s}^2 \text{ directed toward the center of Earth}}.$

P4.29 $r = 0.500 \text{ m};$

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500 \text{ m})}{\frac{60.0 \text{ s}}{200 \text{ rev}}} = 10.47 \text{ m/s} = \boxed{10.5 \text{ m/s}}$$

$$a = \frac{v^2}{R} = \frac{(10.47)^2}{0.5} = \boxed{219 \text{ m/s}^2 \text{ inward}}$$

P4.30 $a_c = \frac{v^2}{r}$
 $v = \sqrt{a_c r} = \sqrt{3(9.8 \text{ m/s}^2)(9.45 \text{ m})} = 16.7 \text{ m/s}$

Each revolution carries the astronaut over a distance of $2\pi r = 2\pi(9.45 \text{ m}) = 59.4 \text{ m}$. Then the rotation rate is

$$16.7 \text{ m/s} \left(\frac{1 \text{ rev}}{59.4 \text{ m}} \right) = \boxed{0.281 \text{ rev/s}}.$$

P4.31 (a) $v = r\omega$
 At 8.00 rev/s , $v = (0.600 \text{ m})(8.00 \text{ rev/s})(2\pi \text{ rad/rev}) = 30.2 \text{ m/s} = 9.60\pi \text{ m/s}$.
 At 6.00 rev/s , $v = (0.900 \text{ m})(6.00 \text{ rev/s})(2\pi \text{ rad/rev}) = 33.9 \text{ m/s} = 10.8\pi \text{ m/s}$.

$\boxed{6.00 \text{ rev/s}}$ gives the larger linear speed.

(b) $\text{Acceleration} = \frac{v^2}{r} = \frac{(9.60\pi \text{ m/s})^2}{0.600 \text{ m}} = \boxed{1.52 \times 10^3 \text{ m/s}^2}.$

(c) At 6.00 rev/s , $\text{acceleration} = \frac{(10.8\pi \text{ m/s})^2}{0.900 \text{ m}} = \boxed{1.28 \times 10^3 \text{ m/s}^2}.$

P4.32 The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration.

$$a_c = g$$

so

$$\frac{v^2}{r} = g.$$

$$\text{Solving for the velocity, } v = \sqrt{rg} = \sqrt{(6,400 + 600)(10^3 \text{ m})(8.21 \text{ m/s}^2)} = \boxed{7.58 \times 10^3 \text{ m/s}}$$

$$v = \frac{2\pi r}{T}$$

and

$$T = \frac{2\pi r}{v} = \frac{2\pi(7,000 \times 10^3 \text{ m})}{7.58 \times 10^3 \text{ m/s}} = \boxed{5.80 \times 10^3 \text{ s}}$$

$$T = 5.80 \times 10^3 \text{ s} \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 96.7 \text{ min.}$$

Section 4.5 Tangential and Radial Acceleration

P4.33 We assume the train is still slowing down at the instant in question.

$$a_c = \frac{v^2}{r} = 1.29 \text{ m/s}^2$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0 \text{ km/h})(10^3 \text{ m/km})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)}{15.0 \text{ s}} = -0.741 \text{ m/s}^2$$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(1.29 \text{ m/s}^2)^2 + (-0.741 \text{ m/s}^2)^2}$$

$$\text{at an angle of } \tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right)$$

$$\mathbf{a} = \boxed{1.48 \text{ m/s}^2 \text{ inward and } 29.9^\circ \text{ backward}}$$

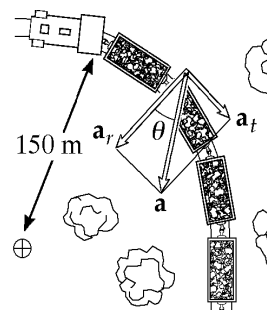


FIG. P4.33

P4.34 (a) $a_t = \boxed{0.600 \text{ m/s}^2}$

(b) $a_r = \frac{v^2}{r} = \frac{(4.00 \text{ m/s})^2}{20.0 \text{ m}} = \boxed{0.800 \text{ m/s}^2}$

(c) $a = \sqrt{a_t^2 + a_r^2} = \boxed{1.00 \text{ m/s}^2}$

$$\theta = \tan^{-1} \frac{a_r}{a_t} = \boxed{53.1^\circ \text{ inward from path}}$$

P4.35 $r = 2.50 \text{ m}$, $a = 15.0 \text{ m/s}^2$

(a) $a_c = a \cos 30.0^\circ = (15.0 \text{ m/s}^2)(\cos 30^\circ) = \boxed{13.0 \text{ m/s}^2}$

(b) $a_c = \frac{v^2}{r}$
 so $v^2 = ra_c = 2.50 \text{ m}(13.0 \text{ m/s}^2) = 32.5 \text{ m}^2/\text{s}^2$
 $v = \sqrt{32.5} \text{ m/s} = \boxed{5.70 \text{ m/s}}$

(c) $a^2 = a_t^2 + a_r^2$
 so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0 \text{ m/s}^2)^2 - (13.0 \text{ m/s}^2)^2} = \boxed{7.50 \text{ m/s}^2}$

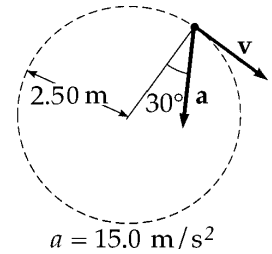


FIG. P4.35

P4.36 (a) See figure to the right.

(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the centripetal acceleration:

$$a_c = (22.5 \text{ m/s}^2)\cos(90.0^\circ - 36.9^\circ) + (20.2 \text{ m/s}^2)\cos 36.9^\circ = \boxed{29.7 \text{ m/s}^2}$$

(c) $a_c = \frac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7 \text{ m/s}^2 (1.50 \text{ m})} = 6.67 \text{ m/s}$ tangent to circle
 $\mathbf{v} = \boxed{6.67 \text{ m/s at } 36.9^\circ \text{ above the horizontal}}$

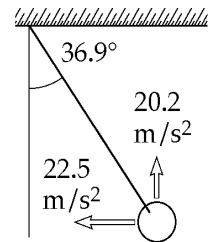


FIG. P4.36

***P4.37** Let i be the starting point and f be one revolution later. The curvilinear motion with constant tangential acceleration is described by

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$

$$2\pi r = 0 + \frac{1}{2}a_t t^2$$

$$a_t = \frac{4\pi r}{t^2}$$

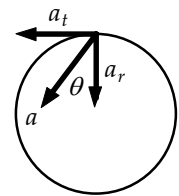


FIG. P4.37

and $v_{xf} = v_{xi} + a_x t$, $v_f = 0 + a_t t = \frac{4\pi r}{t}$. The magnitude of the radial acceleration is $a_r = \frac{v_f^2}{r} = \frac{16\pi^2 r^2}{t^2 r}$.

Then $\tan \theta = \frac{a_t}{a_r} = \frac{4\pi r t^2}{t^2 16\pi^2 r} = \frac{1}{4\pi}$ $\theta = \boxed{4.55^\circ}$.

Section 4.6 Relative Velocity and Relative Acceleration

P4.38 (a) $\mathbf{v}_H = 0 + \mathbf{a}_H t = (3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}) \text{ m/s}^2 (5.00 \text{ s})$
 $\mathbf{v}_H = (15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}}) \text{ m/s}$
 $\mathbf{v}_J = 0 + \mathbf{a}_J t = (1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}) \text{ m/s}^2 (5.00 \text{ s})$
 $\mathbf{v}_J = (5.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}) \text{ m/s}$
 $\mathbf{v}_{HJ} = \mathbf{v}_H - \mathbf{v}_J = (15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 5.00\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}) \text{ m/s}$
 $\mathbf{v}_{HJ} = (10.0\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}) \text{ m/s}$
 $|\mathbf{v}_{HJ}| = \sqrt{(10.0)^2 + (25.0)^2} \text{ m/s} = \boxed{26.9 \text{ m/s}}$

(b) $\mathbf{r}_H = 0 + 0 + \frac{1}{2} \mathbf{a}_H t^2 = \frac{1}{2} (3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}) \text{ m/s}^2 (5.00 \text{ s})^2$
 $\mathbf{r}_H = (37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}) \text{ m}$
 $\mathbf{r}_J = \frac{1}{2} (1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}) \text{ m/s}^2 (5.00 \text{ s})^2 = (12.5\hat{\mathbf{i}} + 37.5\hat{\mathbf{j}}) \text{ m}$
 $\mathbf{r}_{HJ} = \mathbf{r}_H - \mathbf{r}_J = (37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}} - 12.5\hat{\mathbf{i}} - 37.5\hat{\mathbf{j}}) \text{ m}$
 $\mathbf{r}_{HJ} = (25.0\hat{\mathbf{i}} - 62.5\hat{\mathbf{j}}) \text{ m}$
 $|\mathbf{r}_{HJ}| = \sqrt{(25.0)^2 + (62.5)^2} \text{ m} = \boxed{67.3 \text{ m}}$

(c) $\mathbf{a}_{HJ} = \mathbf{a}_H - \mathbf{a}_J = (3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}) \text{ m/s}^2$
 $\mathbf{a}_{HJ} = \boxed{(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}) \text{ m/s}^2}$

- *P4.39** $\mathbf{v}_{ce}$ = the velocity of the car relative to the earth.
 $\mathbf{v}_{wc}$ = the velocity of the water relative to the car.
 $\mathbf{v}_{we}$ = the velocity of the water relative to the earth.

These velocities are related as shown in the diagram at the right.

- (a) Since $\mathbf{v}_{we}$ is vertical, $v_{wc} \sin 60.0^\circ = v_{ce} = 50.0 \text{ km/h}$ or
 $\mathbf{v}_{wc} = \boxed{57.7 \text{ km/h at } 60.0^\circ \text{ west of vertical}}.$

- (b) Since $\mathbf{v}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc} \cos 60.0^\circ = (57.7 \text{ km/h}) \cos 60.0^\circ = \boxed{28.9 \text{ km/h downward}}.$$

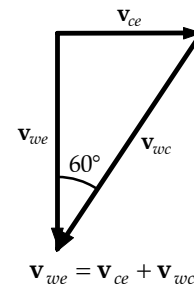


FIG. P4.39

- P4.40** The bumpers are initially $100 \text{ m} = 0.100 \text{ km}$ apart. After time t the bumper of the leading car travels $40.0 t$, while the bumper of the chasing car travels $60.0t$.

Since the cars are side by side at time t , we have

$$0.100 + 40.0t = 60.0t,$$

yielding

$$t = 5.00 \times 10^{-3} \text{ h} = \boxed{18.0 \text{ s}}.$$

- P4.41** Total time in still water $t = \frac{d}{v} = \frac{2000}{1.20} = \boxed{1.67 \times 10^3 \text{ s}}.$

Total time = time upstream plus time downstream:

$$t_{\text{up}} = \frac{1000}{(1.20 - 0.500)} = 1.43 \times 10^3 \text{ s}$$

$$t_{\text{down}} = \frac{1000}{1.20 + 0.500} = 588 \text{ s}.$$

Therefore, $t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3 \text{ s}}.$

- P4.42** $v = \sqrt{150^2 + 30.0^2} = \boxed{153 \text{ km/h}}$
- $\theta = \tan^{-1}\left(\frac{30.0}{150}\right) = \boxed{11.3^\circ \text{ north of west}}$

- P4.43** For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$.

Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{\frac{2L}{c}}{1 - \frac{v^2}{c^2}}}.$$

For Beth, her cross-stream speed (both ways) is

$$\sqrt{c^2 - v^2}.$$

Thus, the total time for Beth is $t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \boxed{\frac{\frac{2L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}}.$

Since $1 - \frac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

- P4.44** (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

$$a = \sqrt{(2.50 \text{ m/s})^2 + (9.80 \text{ m/s})^2} = \boxed{10.1 \text{ m/s}^2}$$

$$\tan \theta = \frac{2.50 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 0.255$$

$$\theta = \boxed{14.3^\circ \text{ to the south from the vertical}}$$

(b) $a = \boxed{9.80 \text{ m/s}^2 \text{ vertically downward}}$

- P4.45** Identify the student as the S' observer and the professor as the S observer. For the initial motion in S' , we have

$$\frac{v'_y}{v'_x} = \tan 60.0^\circ = \sqrt{3}.$$

Let u represent the speed of S' relative to S . Then because there is no x -motion in S , we can write $v_x = v'_x + u = 0$ so that $v'_x = -u = -10.0 \text{ m/s}$. Hence the ball is thrown backwards in S' . Then,

$$v_y = v'_y = \sqrt{3} |v'_x| = 10.0\sqrt{3} \text{ m/s}.$$

Using $v_y^2 = 2gh$ we find

$$h = \frac{(10.0\sqrt{3} \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{15.3 \text{ m}}.$$

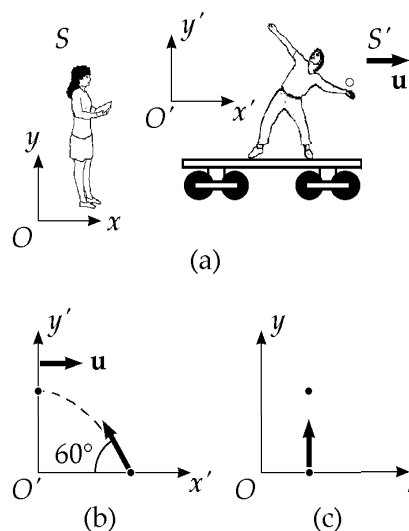


FIG. P4.45

The motion of the ball as seen by the student in S' is shown in diagram (b). The view of the professor in S is shown in diagram (c).

- *P4.46** Choose the x -axis along the 20-km distance. The y -components of the displacements of the ship and the speedboat must agree:

$$(26 \text{ km/h})t \sin(40^\circ - 15^\circ) = (50 \text{ km/h})t \sin \alpha$$

$$\alpha = \sin^{-1} \frac{11.0}{50} = 12.7^\circ.$$

The speedboat should head

$$15^\circ + 12.7^\circ = \boxed{27.7^\circ \text{ east of north}}.$$

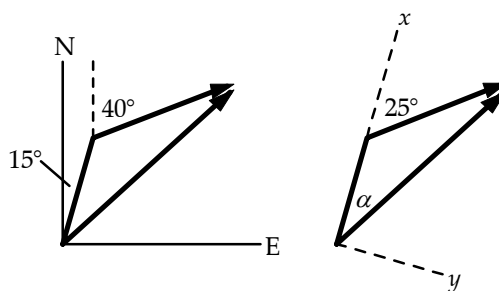


FIG. P4.46

Additional Problems

***P4.47** (a) The speed at the top is $v_x = v_i \cos \theta_i = (143 \text{ m/s}) \cos 45^\circ = \boxed{101 \text{ m/s}}$.

(b) In free fall the plane reaches altitude given by

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y(y_f - y_i) \\ 0 &= (143 \text{ m/s} \sin 45^\circ)^2 + 2(-9.8 \text{ m/s}^2)(y_f - 31\,000 \text{ ft}) \\ y_f &= 31\,000 \text{ ft} + 522 \text{ m} \left(\frac{3.28 \text{ ft}}{1 \text{ m}} \right) = \boxed{3.27 \times 10^3 \text{ ft}}. \end{aligned}$$

(c) For the whole free fall motion $v_{yf} = v_{yi} + a_y t$

$$\begin{aligned} -101 \text{ m/s} &= +101 \text{ m/s} - (9.8 \text{ m/s}^2)t \\ t &= \boxed{20.6 \text{ s}} \end{aligned}$$

(d) $a_c = \frac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{0.8(9.8 \text{ m/s}^2)4,130 \text{ m}} = \boxed{180 \text{ m/s}}$$

P4.48 At any time t , the two drops have identical y -coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2|x(t)| = 2(v_{xi}t) = 2(v_i \cos \theta_i)t = \boxed{2v_i t \cos \theta_i}.$$

P4.49 After the string breaks the ball is a projectile, and reaches the ground at time t : $y_f = v_{yi}t + \frac{1}{2}a_y t^2$

$$-1.20 \text{ m} = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

so $t = 0.495 \text{ s}$.

Its constant horizontal speed is $v_x = \frac{x}{t} = \frac{2.00 \text{ m}}{0.495 \text{ s}} = 4.04 \text{ m/s}$

so before the string breaks $a_c = \frac{v_x^2}{r} = \frac{(4.04 \text{ m/s})^2}{0.300 \text{ m}} = \boxed{54.4 \text{ m/s}^2}$.

P4.50 (a) $y_f = (\tan \theta_i)(x_f) - \frac{g}{2v_i^2 \cos^2 \theta_i} x_f^2$

Setting $x_f = d \cos \phi$, and $y_f = d \sin \phi$, we have

$$d \sin \phi = (\tan \theta_i)(d \cos \phi) - \frac{g}{2v_i^2 \cos^2 \theta_i} (d \cos \phi)^2.$$

Solving for d yields, $d = \frac{2v_i^2 \cos \theta_i [\sin \theta_i \cos \phi - \sin \phi \cos \theta_i]}{g \cos^2 \phi}$

or $d = \boxed{\frac{2v_i^2 \cos \theta_i \sin(\theta_i - \phi)}{g \cos^2 \phi}}.$

(b) Setting $\frac{dd}{d\theta_i} = 0$ leads to $\boxed{\theta_i = 45^\circ + \frac{\phi}{2}}$ and $\boxed{d_{\max} = \frac{v_i^2 (1 - \sin \phi)}{g \cos^2 \phi}}.$

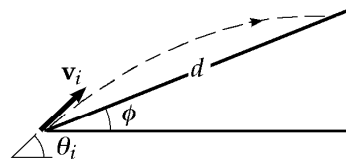


FIG. P4.50

P4.51 Refer to the sketch:

(b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i \cos 35.0^\circ)t$.

$\Delta y = v_{yi}t + \frac{1}{2}at^2$; substitution yields

$$20.0 = (v_i \sin 35.0^\circ)t + \frac{1}{2}(-9.80)t^2.$$

Solving the above gives $t = \boxed{3.81 \text{ s}}.$

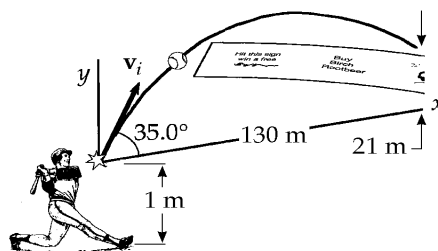


FIG. P4.51

(a) $v_i = \boxed{41.7 \text{ m/s}}$

(c) $v_{yf} = v_i \sin \theta_i - gt$, $v_x = v_i \cos \theta_i$

At $t = 3.81 \text{ s}$, $v_{yf} = 41.7 \sin 35.0^\circ - (9.80)(3.81) = \boxed{-13.4 \text{ m/s}}$

$$v_x = (41.7 \cos 35.0^\circ) = \boxed{34.1 \text{ m/s}}$$

$$v_f = \sqrt{v_x^2 + v_{yf}^2} = \boxed{36.7 \text{ m/s}}.$$

- P4.52** (a) The moon's gravitational acceleration is the probe's centripetal acceleration:
(For the moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$

$$\frac{1}{6}(9.80 \text{ m/s}^2) = \frac{v^2}{1.74 \times 10^6 \text{ m}}$$

$$v = \sqrt{2.84 \times 10^6 \text{ m}^2/\text{s}^2} = \boxed{1.69 \text{ km/s}}$$

(b) $v = \frac{2\pi r}{T}$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74 \times 10^6 \text{ m})}{1.69 \times 10^3 \text{ m/s}} = 6.47 \times 10^3 \text{ s} = \boxed{1.80 \text{ h}}$$

P4.53 (a) $a_c = \frac{v^2}{r} = \frac{(5.00 \text{ m/s})^2}{1.00 \text{ m}} = \boxed{25.0 \text{ m/s}^2}$

$$a_t = g = \boxed{9.80 \text{ m/s}^2}$$

- (b) See figure to the right.

(c) $a = \sqrt{a_c^2 + a_t^2} = \sqrt{(25.0 \text{ m/s}^2)^2 + (9.80 \text{ m/s}^2)^2} = \boxed{26.8 \text{ m/s}^2}$

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\frac{9.80 \text{ m/s}^2}{25.0 \text{ m/s}^2} = \boxed{21.4^\circ}$$

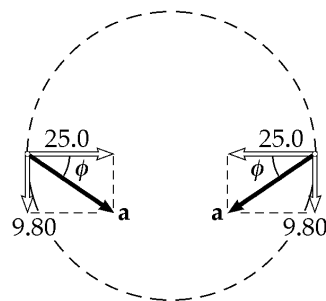


FIG. P4.53

P4.54 $x_f = v_{ix}t = v_i t \cos 40.0^\circ$

Thus, when $x_f = 10.0 \text{ m}$, $t = \frac{10.0 \text{ m}}{v_i \cos 40.0^\circ}$.

At this time, y_f should be $3.05 \text{ m} - 2.00 \text{ m} = 1.05 \text{ m}$.

Thus, $1.05 \text{ m} = \frac{(v_i \sin 40.0^\circ)10.0 \text{ m}}{v_i \cos 40.0^\circ} + \frac{1}{2}(-9.80 \text{ m/s}^2)\left[\frac{10.0 \text{ m}}{v_i \cos 40.0^\circ}\right]^2$.

From this, $v_i = \boxed{10.7 \text{ m/s}}$.

104 Motion in Two Dimensions

- P4.55** The special conditions allowing use of the horizontal range equation applies.
For the ball thrown at 45° ,

$$D = R_{45} = \frac{v_i^2 \sin 90}{g}.$$

For the bouncing ball,

$$D = R_1 + R_2 = \frac{v_i^2 \sin 2\theta}{g} + \frac{\left(\frac{v_i}{2}\right)^2 \sin 2\theta}{g}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

- (a) We require:

$$\begin{aligned} \frac{v_i^2}{g} &= \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g} \\ \sin 2\theta &= \frac{4}{5} \\ \theta &= 26.6^\circ \end{aligned}$$

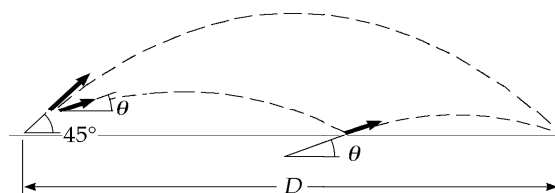


FIG. P4.55

- (b) The time for any symmetric parabolic flight is given by

$$\begin{aligned} y_f &= v_{yi}t - \frac{1}{2}gt^2 \\ 0 &= v_i \sin \theta_i t - \frac{1}{2}gt^2. \end{aligned}$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin \theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0°

$$t_{45} = \frac{2v_i \sin 45.0^\circ}{g}.$$

For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6^\circ}{g} + \frac{2\left(\frac{v_i}{2}\right) \sin 26.6^\circ}{g} = \frac{3v_i \sin 26.6^\circ}{g}.$$

The ratio of this time to that for no bounce is

$$\frac{\frac{3v_i \sin 26.6^\circ}{g}}{\frac{2v_i \sin 45.0^\circ}{g}} = \frac{1.34}{1.41} = \boxed{0.949}.$$

P4.56 Using the range equation (Equation 4.14)

$$R = \frac{v_i^2 \sin(2\theta_i)}{g}$$

the maximum range occurs when $\theta_i = 45^\circ$, and has a value $R = \frac{v_i^2}{g}$. Given R , this yields $v_i = \sqrt{gR}$.

If the boy uses the same speed to throw the ball vertically upward, then

$$v_y = \sqrt{gR} - gt \text{ and } y = \sqrt{gR}t - \frac{gt^2}{2}$$

at any time, t .

At the maximum height, $v_y = 0$, giving $t = \sqrt{\frac{R}{g}}$, and so the maximum height reached is

$$y_{\max} = \sqrt{gR} \sqrt{\frac{R}{g}} - \frac{g}{2} \left(\sqrt{\frac{R}{g}} \right)^2 = R - \frac{R}{2} = \boxed{\frac{R}{2}}.$$

P4.57 Choose upward as the positive y -direction and leftward as the positive x -direction. The vertical height of the stone when released from A or B is

$$y_i = (1.50 + 1.20 \sin 30.0^\circ) \text{ m} = 2.10 \text{ m}$$

(a) The equations of motion after release at A are

$$v_y = v_i \sin 60.0^\circ - gt = (1.30 - 9.80t) \text{ m/s}$$

$$v_x = v_i \cos 60.0^\circ = 0.750 \text{ m/s}$$

$$y = (2.10 + 1.30t - 4.90t^2) \text{ m}$$

$$\Delta x_A = (0.750t) \text{ m}$$

$$\text{When } y = 0, t = \frac{-1.30 \pm \sqrt{(1.30)^2 + 41.2}}{-9.80} = 0.800 \text{ s. Then, } \Delta x_A = (0.750)(0.800) \text{ m} = \boxed{0.600 \text{ m}}.$$

(b) The equations of motion after release at point B are

$$v_y = v_i(-\sin 60.0^\circ) - gt = (-1.30 - 9.80t) \text{ m/s}$$

$$v_x = v_i \cos 60.0^\circ = 0.750 \text{ m/s}$$

$$y = (2.10 - 1.30t - 4.90t^2) \text{ m.}$$

$$\text{When } y = 0, t = \frac{+1.30 \pm \sqrt{(-1.30)^2 + 41.2}}{-9.80} = 0.536 \text{ s. Then, } \Delta x_B = (0.750)(0.536) \text{ m} = \boxed{0.402 \text{ m}}.$$

$$(c) \quad a_r = \frac{v^2}{r} = \frac{(1.50 \text{ m/s})^2}{1.20 \text{ m}} = \boxed{1.87 \text{ m/s}^2 \text{ toward the center}}$$

$$(d) \quad \text{After release, } \mathbf{a} = -g \hat{\mathbf{j}} = \boxed{9.80 \text{ m/s}^2 \text{ downward}}$$

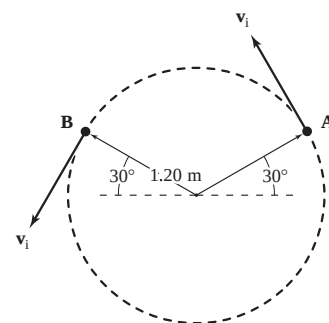


FIG. P4.57

P4.58 The football travels a horizontal distance

$$R = \frac{v_i^2 \sin(2\theta_i)}{g} = \frac{(20.0)^2 \sin(60.0^\circ)}{9.80} = 35.3 \text{ m}.$$

Time of flight of ball is

$$t = \frac{2v_i \sin \theta_i}{g} = \frac{2(20.0) \sin 30.0^\circ}{9.80} = 2.04 \text{ s}.$$

The receiver is Δx away from where the ball lands and $\Delta x = 35.3 - 20.0 = 15.3 \text{ m}$. To cover this distance in 2.04 s, he travels with a velocity

$$v = \frac{15.3}{2.04} = \boxed{7.50 \text{ m/s in the direction the ball was thrown}}.$$

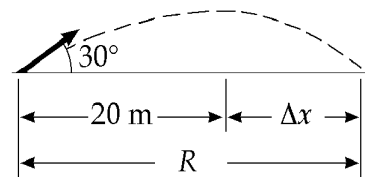


FIG. P4.58

P4.59 (a) $\Delta y = -\frac{1}{2}gt^2$; $\Delta x = v_i t$

Combine the equations eliminating t :

$$\Delta y = -\frac{1}{2}g\left(\frac{\Delta x}{v_i}\right)^2.$$

$$\text{From this, } (\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right)v_i^2$$

$$\text{thus } \Delta x = v_i \sqrt{\frac{-2\Delta y}{g}} = 275 \sqrt{\frac{-2(-300)}{9.80}} = 6.80 \times 10^3 = \boxed{6.80 \text{ km}}.$$

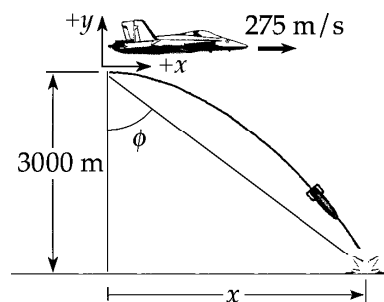


FIG. P4.59

(b) The plane has the same velocity as the bomb in the x direction. Therefore, the plane will be $\boxed{3\,000 \text{ m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan \phi = \frac{\Delta x}{\Delta y}$

$$\text{therefore, } \phi = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6\,800}{3\,000}\right) = \boxed{66.2^\circ}.$$

- *P4.60** (a) We use the approximation mentioned in the problem. The time to travel 200 m horizontally is
- $$t = \frac{\Delta x}{v_x} = \frac{200 \text{ m}}{1,000 \text{ m/s}} = 0.200 \text{ s. The bullet falls by}$$

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.8 \text{ m/s}^2)(0.2 \text{ s})^2 = \boxed{-0.196 \text{ m}}.$$

- (b) The telescope axis must point below the barrel axis
by $\theta = \tan^{-1} \frac{0.196 \text{ m}}{200 \text{ m}} = \boxed{0.0561^\circ}$.

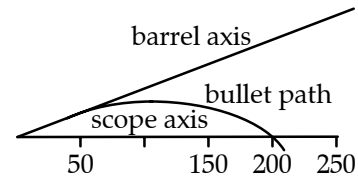


FIG. P4.60(b)

- (c) $t = \frac{50.0 \text{ m}}{1\,000 \text{ m/s}} = 0.0500 \text{ s}$. The bullet falls by only

$$\Delta y = \frac{1}{2}(-9.8 \text{ m/s}^2)(0.05 \text{ s})^2 = -0.0122 \text{ m}.$$

At range 50 m = $\frac{1}{4}(200 \text{ m})$, the scope axis points to a location $\frac{1}{4}(19.6 \text{ cm}) = 4.90 \text{ cm}$ above the barrel axis, so the sharpshooter must **aim low** by $4.90 \text{ cm} - 1.22 \text{ cm} = \boxed{3.68 \text{ cm}}$.

- (d) $t = \frac{150 \text{ m}}{1\,000 \text{ m/s}} = 0.150 \text{ s}$

$$\Delta y = \frac{1}{2}(-9.8 \text{ m/s}^2)(0.15 \text{ s})^2 = 0.110 \text{ m}$$

$$\boxed{\text{Aim low}} \text{ by } \frac{150}{200}(19.6 \text{ cm}) - 11.0 \text{ cm} = \boxed{3.68 \text{ cm}}.$$

- (e) $t = \frac{250 \text{ m}}{1\,000 \text{ m/s}} = 0.250 \text{ s}$

$$\Delta y = \frac{1}{2}(-9.8 \text{ m/s}^2)(0.25 \text{ s})^2 = 0.306 \text{ m}$$

$$\boxed{\text{Aim high}} \text{ by } 30.6 \text{ cm} - \frac{250}{200}(19.6 \text{ cm}) = \boxed{6.12 \text{ cm}}.$$

- (f), (g) Many marksmen have a hard time believing it, but they should aim low in both cases. As in case (a) above, the time of flight is very nearly 0.200 s and the bullet falls below the barrel axis by 19.6 cm on its way. The 0.0561° angle would cut off a 19.6-cm distance on a vertical wall at a horizontal distance of 200 m, but on a vertical wall up at 30° it cuts off distance h as shown, where $\cos 30^\circ = 19.6 \text{ cm}/h$, $h = 22.6 \text{ cm}$. The marksman must **aim low** by $22.6 \text{ cm} - 19.6 \text{ cm} = 3.03 \text{ cm}$. The answer can be obtained by considering limiting cases. Suppose the target is nearly straight above or below you. Then gravity will not cause deviation of the path of the bullet, and one must aim low as in part (c) to cancel out the sighting-in of the telescope.

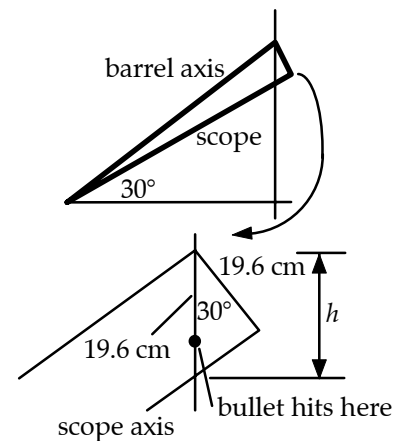


FIG. P4.60(f-g)

- P4.61** (a) From Part (c), the raptor dives for $6.34 - 2.00 = 4.34$ s undergoing displacement 197 m downward and $(10.0)(4.34) = 43.4$ m forward.

$$v = \frac{\Delta d}{\Delta t} \sqrt{(197)^2 + (43.4)^2} = \boxed{46.5 \text{ m/s}}$$

(b) $\alpha = \tan^{-1}\left(\frac{-197}{43.4}\right) = \boxed{-77.6^\circ}$

(c) $197 = \frac{1}{2}gt^2, \boxed{t = 6.34 \text{ s}}$

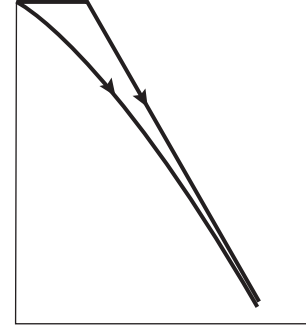


FIG. P4.61

- P4.62** Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - \frac{gx^2}{2v_i^2}$.

- (a) The elevation y_r of points on the rock is described by

$$y_r^2 + x^2 = R^2.$$

We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$

$$\begin{aligned} \left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 &> R^2 \\ R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 &> R^2 \\ \frac{g^2x^4}{4v_i^4} + x^2 &> \frac{gx^2R}{v_i^2}. \end{aligned}$$

If this inequality is satisfied for x approaching zero, it will be true for all x . If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock: $1 > \frac{gR}{v_i^2}$

$$\boxed{v_i > \sqrt{gR}}.$$

- (b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \frac{gx^2}{2gR}$
or $x = R\sqrt{2}$.

The distance from the rock's base is

$$x - R = \boxed{(\sqrt{2} - 1)R}.$$

P4.63 (a) While on the incline

$$\begin{aligned}
 v_f^2 - v_i^2 &= 2a\Delta x \\
 v_f - v_i &= at \\
 v_f^2 - 0 &= 2(4.00)(50.0) \\
 20.0 - 0 &= 4.00t \\
 v_f &= \boxed{20.0 \text{ m/s}} \\
 t &= \boxed{5.00 \text{ s}}
 \end{aligned}$$

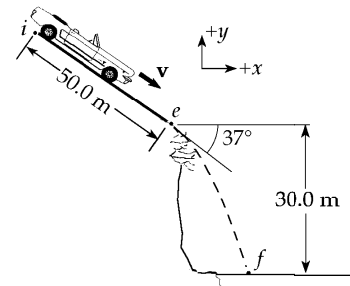


FIG. P4.63

(b) Initial free-flight conditions give us

$$v_{xi} = 20.0 \cos 37.0^\circ = 16.0 \text{ m/s}$$

and

$$v_{yi} = -20.0 \sin 37.0^\circ = -12.0 \text{ m/s}$$

$$v_{xf} = v_{xi} \text{ since } a_x = 0$$

$$v_{yf} = -\sqrt{2a_y\Delta y + v_{yi}^2} = -\sqrt{2(-9.80)(-30.0) + (-12.0)^2} = -27.1 \text{ m/s}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(16.0)^2 + (-27.1)^2} = \boxed{31.5 \text{ m/s at } 59.4^\circ \text{ below the horizontal}}$$

$$(c) \quad t_1 = 5 \text{ s}; \quad t_2 = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-27.1 + 12.0}{-9.80} = 1.53 \text{ s}$$

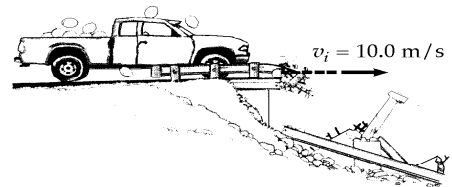
$$t = t_1 + t_2 = \boxed{6.53 \text{ s}}$$

$$(d) \quad \Delta x = v_{xi}t_1 = 16.0(1.53) = \boxed{24.5 \text{ m}}$$

P4.64 Equation of bank: $y^2 = 16x$ (1)

Equations of motion: $x = v_i t$ (2)

$$y = -\frac{1}{2}gt^2 \quad (3)$$



Substitute for t from (2) into (3) $y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)$. Equate y

from the bank equation to y from the equations of motion:

FIG. P4.64

$$16x = \left[-\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right) \right]^2 \Rightarrow \frac{g^2 x^4}{4v_i^4} - 16x = x\left(\frac{g^2 x^3}{4v_i^4} - 16\right) = 0.$$

From this, $x = 0$ or $x^3 = \frac{64v_i^4}{g^2}$ and $x = 4\left(\frac{10^4}{9.80^2}\right)^{1/3} = \boxed{18.8 \text{ m}}$. Also,

$$y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right) = -\frac{1}{2}\frac{(9.80)(18.8)^2}{(10.0)^2} = \boxed{-17.3 \text{ m}}.$$

P4.65 (a) Coyote: $\Delta x = \frac{1}{2}at^2$; $70.0 = \frac{1}{2}(15.0)t^2$
 Roadrunner: $\Delta x = v_i t$; $70.0 = v_i t$

Solving the above, we get

$$v_i = \boxed{22.9 \text{ m/s}} \text{ and } t = 3.06 \text{ s.}$$

(b) At the edge of the cliff,

$$v_{xi} = at = (15.0)(3.06) = 45.8 \text{ m/s.}$$

Substituting into $\Delta y = \frac{1}{2}a_y t^2$, we find

$$\begin{aligned} -100 &= \frac{1}{2}(-9.80)t^2 \\ t &= 4.52 \text{ s} \\ \Delta x &= v_{xi}t + \frac{1}{2}a_x t^2 = (45.8)(4.52 \text{ s}) + \frac{1}{2}(15.0)(4.52 \text{ s})^2. \end{aligned}$$

Solving,

$$\Delta x = \boxed{360 \text{ m}}.$$

(c) For the Coyote's motion through the air

$$\begin{aligned} v_{xf} &= v_{xi} + a_x t = 45.8 + 15(4.52) = \boxed{114 \text{ m/s}} \\ v_{yf} &= v_{yi} + a_y t = 0 - 9.80(4.52) = \boxed{-44.3 \text{ m/s}}. \end{aligned}$$

P4.66 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6 \text{ m})}{0.1 \text{ s}} \cong 9 \text{ m/s}$$

and its centripetal acceleration is $\frac{v^2}{r} \cong \frac{(9 \text{ m/s})^2}{0.6 \text{ m}} \boxed{\sim 10^2 \text{ m/s}^2}.$

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

P4.67 (a) $\Delta x = v_{xi}t, \Delta y = v_{yi}t + \frac{1}{2}gt^2$

$$d \cos 50.0^\circ = (10.0 \cos 15.0^\circ)t$$

and

$$-d \sin 50.0^\circ = (10.0 \sin 15.0^\circ)t + \frac{1}{2}(-9.80)t^2.$$

Solving, $d = \boxed{43.2 \text{ m}}$ and $t = 2.88 \text{ s}$.

(b) Since $a_x = 0$,

$$v_{xf} = v_{xi} = 10.0 \cos 15.0^\circ = \boxed{9.66 \text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t = 10.0 \sin 15.0^\circ - 9.80(2.88) = \boxed{-25.6 \text{ m/s}}.$$

Air resistance would decrease the values of the range and maximum height. As an airfoil, he can get some lift and increase his distance.

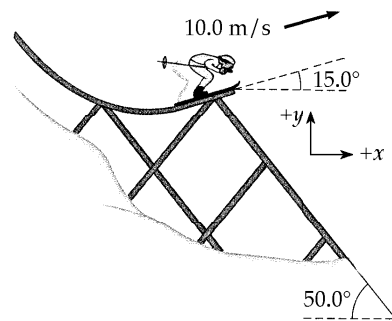


FIG. P4.67

***P4.68** For one electron, we have

$$y = v_{iy}t, D = v_{ix}t + \frac{1}{2}a_x t^2 \cong \frac{1}{2}a_x t^2, v_{yf} = v_{yi}, \text{ and } v_{xf} = v_{xi} + a_x t \cong a_x t.$$

The angle its direction makes with the x -axis is given by

$$\theta = \tan^{-1} \frac{v_{yf}}{v_{xf}} = \tan^{-1} \frac{v_{yi}}{a_x t} = \tan^{-1} \frac{v_{yi}t}{a_x t^2} = \tan^{-1} \frac{y}{2D}.$$

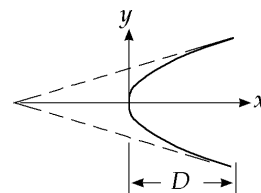


FIG. P4.68

Thus the horizontal distance from the aperture to the virtual source is $2D$. The source is at coordinate $\boxed{x = -D}$.

***P4.69** (a) The ice chest floats downstream 2 km in time t , so that $2 \text{ km} = v_w t$. The upstream motion of the boat is described by $d = (v - v_w)15 \text{ min}$. The downstream motion is described by $d + 2 \text{ km} = (v + v_w)(t - 15 \text{ min})$. We eliminate $t = \frac{2 \text{ km}}{v_w}$ and d by substitution:

$$(v - v_w)15 \text{ min} + 2 \text{ km} = (v + v_w) \left(\frac{2 \text{ km}}{v_w} - 15 \text{ min} \right)$$

$$v(15 \text{ min}) - v_w(15 \text{ min}) + 2 \text{ km} = \frac{v}{v_w} 2 \text{ km} + 2 \text{ km} - v(15 \text{ min}) - v_w(15 \text{ min})$$

$$v(30 \text{ min}) = \frac{v}{v_w} 2 \text{ km}$$

$$v_w = \frac{2 \text{ km}}{30 \text{ min}} = \boxed{4.00 \text{ km/h}}.$$

(b) In the reference frame of the water, the chest is motionless. The boat travels upstream for 15 min at speed v , and then downstream at the same speed, to return to the same point. Thus it travels for 30 min. During this time, the falls approach the chest at speed v_w , traveling 2 km. Thus

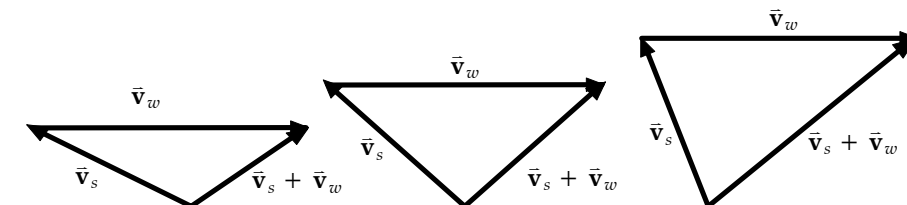
$$v_w = \frac{\Delta x}{\Delta t} = \frac{2 \text{ km}}{30 \text{ min}} = \boxed{4.00 \text{ km/h}}.$$

*P4.70 Let the river flow in the x direction.

- (a) To minimize time, swim perpendicular to the banks in the y direction. You are in the water for time t in $\Delta y = v_y t$, $t = \frac{80 \text{ m}}{1.5 \text{ m/s}} = 53.3 \text{ s}$.

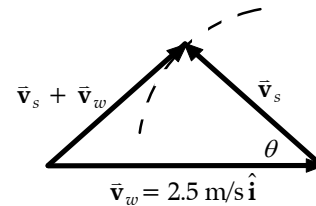
- (b) The water carries you downstream by $\Delta x = v_x t = (2.50 \text{ m/s})53.3 \text{ s} = \boxed{133 \text{ m}}$.

- (c)



To minimize downstream drift, you should swim so that your resultant velocity $\vec{v}_s + \vec{v}_w$ is perpendicular to your swimming velocity $\vec{v}_s$ relative to the water. This condition is shown in the middle picture. It maximizes the angle between the resultant velocity and the shore. The angle between $\vec{v}_s$ and the shore is given by $\cos \theta = \frac{1.5 \text{ m/s}}{2.5 \text{ m/s}}$,

$$\boxed{\theta = 53.1^\circ}.$$



- (d) Now $v_y = v_s \sin \theta = 1.5 \text{ m/s} \sin 53.1^\circ = 1.20 \text{ m/s}$

$$t = \frac{\Delta y}{v_y} = \frac{80 \text{ m}}{1.2 \text{ m/s}} = 66.7 \text{ s}$$

$$\Delta x = v_x t = (2.5 \text{ m/s} - 1.5 \text{ m/s} \cos 53.1^\circ)66.7 \text{ s} = \boxed{107 \text{ m}}.$$

- *P4.71** Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are $> 45^\circ$; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$

$$x_f = v_{xi}t = v_i(\cos\theta)t$$

Thus

$$t = \frac{x_f}{v_i \cos\theta}.$$

Substitute into the expression for y_f

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i \cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i \cos\theta}\right)^2 = x_f \tan\theta - \frac{gx_f^2}{2v_i^2 \cos^2\theta}$$

but $\frac{1}{\cos^2\theta} = \tan^2\theta + 1$ so $y_f = x_f \tan\theta - \frac{gx_f^2}{2v_i^2}(\tan^2\theta + 1)$ and

$$0 = \frac{gx_f^2}{2v_i^2} \tan^2\theta - x_f \tan\theta + \frac{gx_f^2}{2v_i^2} + y_f.$$

Substitute values, use the quadratic formula and find

$$\tan\theta = 3.905 \text{ or } 1.197, \text{ which gives } \theta_H = 75.6^\circ \text{ and } \theta_L = 50.1^\circ.$$

$$\text{Range (at } \theta_H) = \frac{v_i^2 \sin 2\theta_H}{g} = 3.07 \times 10^3 \text{ m from enemy ship}$$

$$3.07 \times 10^3 - 2500 - 300 = 270 \text{ m from shore.}$$

$$\text{Range (at } \theta_L) = \frac{v_i^2 \sin 2\theta_L}{g} = 6.28 \times 10^3 \text{ m from enemy ship}$$

$$6.28 \times 10^3 - 2500 - 300 = 3.48 \times 10^3 \text{ from shore.}$$

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48 \times 10^3 \text{ m}}$ from the shore.

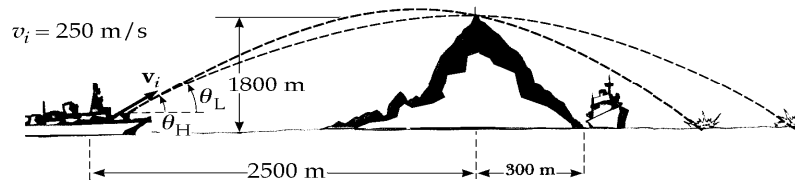


FIG. P4.71

***P4.72** We follow the steps outlined in Example 4.7, eliminating $t = \frac{d \cos \phi}{v_i \cos \theta}$ to find

$$\frac{v_i \sin \theta d \cos \phi}{v_i \cos \theta} - \frac{g d^2 \cos^2 \phi}{2 v_i^2 \cos^2 \theta} = -d \sin \phi.$$

Clearing of fractions,

$$2 v_i^2 \cos \theta \sin \theta \cos \phi - g d \cos^2 \phi = -2 v_i^2 \cos^2 \theta \sin \phi.$$

To maximize d as a function of θ , we differentiate through with respect to θ and set $\frac{dd}{d\theta} = 0$:

$$2 v_i^2 \cos \theta \cos \theta \cos \phi + 2 v_i^2 \sin \theta (-\sin \theta) \cos \phi - g \frac{dd}{d\theta} \cos^2 \phi = -2 v_i^2 2 \cos \theta (-\sin \theta) \sin \phi.$$

We use the trigonometric identities from Appendix B4 $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ and $\sin 2\theta = 2 \sin \theta \cos \theta$ to find $\cos \phi \cos 2\theta = \sin 2\theta \sin \phi$. Next, $\frac{\sin \phi}{\cos \phi} = \tan \phi$ and $\cot 2\theta = \frac{1}{\tan 2\theta}$ give $\cot 2\phi = \tan \phi = \tan(90^\circ - 2\theta)$ so $\phi = 90^\circ - 2\theta$ and $\theta = 45^\circ - \frac{\phi}{2}$.

ANSWERS TO EVEN PROBLEMS

- P4.2** (a) $\mathbf{r} = 18.0t\hat{\mathbf{i}} + (4.00t - 4.90t^2)\hat{\mathbf{j}}$;
 (b) $\mathbf{v} = 18.0\hat{\mathbf{i}} + (4.00 - 9.80t)\hat{\mathbf{j}}$;
 (c) $\mathbf{a} = (-9.80 \text{ m/s}^2)\hat{\mathbf{j}}$;
 (d) $(54.0 \text{ m})\hat{\mathbf{i}} - (32.1 \text{ m})\hat{\mathbf{j}}$;
 (e) $(18.0 \text{ m/s})\hat{\mathbf{i}} - (25.4 \text{ m/s})\hat{\mathbf{j}}$;
 (f) $(-9.80 \text{ m/s}^2)\hat{\mathbf{j}}$

- P4.4** (a) $\mathbf{v} = (-5.00\omega\hat{\mathbf{i}} + 0\hat{\mathbf{j}}) \text{ m/s}$;
 $\mathbf{a} = (0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}) \text{ m/s}^2$;
 (b) $\mathbf{r} = 4.00 \text{ m } \hat{\mathbf{j}}$
 $+ 5.00 \text{ m}(-\sin \omega t \hat{\mathbf{i}} - \cos \omega t \hat{\mathbf{j}})$;
 $\mathbf{v} = 5.00 \text{ m } \omega(-\cos \omega t \hat{\mathbf{i}} + \sin \omega t \hat{\mathbf{j}})$;
 $\mathbf{a} = 5.00 \text{ m } \omega^2(\sin \omega t \hat{\mathbf{i}} + \cos \omega t \hat{\mathbf{j}})$;
 (c) a circle of radius 5.00 m centered at (0, 4.00 m)

- P4.6** (a) $\mathbf{v} = -12.0t\hat{\mathbf{j}} \text{ m/s}$; $\mathbf{a} = -12.0\hat{\mathbf{j}} \text{ m/s}^2$;
 (b) $\mathbf{r} = (3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}) \text{ m}$; $\mathbf{v} = -12.0\hat{\mathbf{j}} \text{ m/s}$

- P4.8** (a) $\mathbf{r} = (5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}}) \text{ m}$;
 $\mathbf{v} = (5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}) \text{ m/s}$;
 (b) $\mathbf{r} = (10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}) \text{ m}$; 7.81 m/s

- P4.10** $(7.23 \times 10^3 \text{ m}, 1.68 \times 10^3 \text{ m})$

- P4.12** (a) $d\sqrt{\frac{g}{2h}}$ horizontally;
 (b) $\tan^{-1}\left(\frac{2h}{d}\right)$ below the horizontal

- P4.14** 0.600 m/s²

- P4.16** (a) 76.0°; (b) the same; (c) $\frac{17d}{8}$

- P4.18** 25.8 m/s

- P4.20** $d \tan \theta_i - \frac{gd^2}{(2v_i^2 \cos^2 \theta_i)}$

- P4.22** 33.5° below the horizontal
- P4.24** (a) 0.852 s; (b) 3.29 m/s; (c) 4.03 m/s; (d) 50.8°; (e) 1.12 s
- P4.26** $\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)$
- P4.28** 0.033 7 m/s² toward the center of the Earth
- P4.30** 0.281 rev/s
- P4.32** 7.58×10^3 m/s; 5.80×10^3 s
- P4.34** (a) 0.600 m/s² forward; (b) 0.800 m/s² inward; (c) 1.00 m/s² forward and 53.1° inward
- P4.36** (a) see the solution; (b) 29.7 m/s²; (c) 6.67 m/s at 36.9° above the horizontal
- P4.38** (a) 26.9 m/s; (b) 67.3 m; (c) $(2.00\hat{i} - 5.00\hat{j})$ m/s²
- P4.40** 18.0 s
- P4.42** 153 km/h at 11.3° north of west
- P4.44** (a) 10.1 m/s² at 14.3° south from the vertical; (b) 9.80 m/s² vertically downward
- P4.46** 27.7° east of north
- P4.48** $2v_i t \cos \theta_i$
- P4.50** (a) see the solution; (b) $\theta_i = 45^\circ + \frac{\phi}{2}$; $d_{\max} = \frac{v_i^2(1 - \sin \phi)}{g \cos^2 \phi}$
- P4.52** (a) 1.69 km/s; (b) 6.47×10^3 s
- P4.54** 10.7 m/s
- P4.56** $\frac{R}{2}$
- P4.58** 7.50 m/s in the direction the ball was thrown
- P4.60** (a) 19.6 cm; (b) 0.0561°; (c) aim low 3.68 cm; (d) aim low 3.68 cm; (e) aim high 6.12 cm; (f) aim low; (g) aim low
- P4.62** (a) $\sqrt{gR}$; (b) $(\sqrt{2} - 1)R$
- P4.64** (18.8 m; -17.3 m)
- P4.66** see the solution; $\sim 10^2$ m/s²
- P4.68** $x = -D$
- P4.70** (a) at 90° to the bank; (b) 133 m; (c) upstream at 53.1° to the bank; (d) 107 m
- P4.72** see the solution